Jet Flap Characteristics for High-Aspect-Ratio Wings

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The thrust hypothesis, its concept and its experimental verification, is discussed in this paper "Characteristics" for jet-flapped wings at zero and nonzero angle of attack, respectively, are also presented Any combination of jet flap parameters (such as jet deflection angle, rate of blowing, and angle of attack) which produces a desired lift may be read directly from such characteristics Furthermore, the characteristics convey data on the economy and efficiency of lift production An "operating line" can be added to the characteristics, if lift production is optimized with respect to jet blowing rates and wing drag Logically, the range of most economical jet flap operation coincides with that portion of the characteristics over which any increment in jet flap momentum causes exactly the same increment in propulsive (measured) thrust Over this portion, characteristics can be artificially constructed or supplemented, if three "constants," relating the total jet flap drag to the total lift, are known Moreover, the application of the jet flap principle to STOL aircraft design is discussed

Nomenclature

J = jet momentum ($=M\ V_J$)
q = dynamic head pressure of freestream
a_0 = constant
c_μ = jet momentum coefficient ($=J/q\ S_w$)
M = mass flow
V_0 = freestream velocity
V_J = jet flow velocity
S_W = wing area
C_1 = constant
C_D' = drag coefficient of wing without blowing
C_{DT} = total drag coefficient of jet flapped wing
C_L' = lift coefficient of wing without blowing
C_{LT} = total lift coefficient of jet-flapped wing
C_{LJ} = jet-induced pressure lift coefficient
C_{LP} = pressure lift coefficient
θ = jet deflection angle
α = angle of attack
C_T = total thrust force coefficient
C_{TM} = total measured thrust as measured with a balance
C_{TP_i} = ideal jet-induced pressure thrust
C_{DP}' = profile drag coefficient for wing without blowing
C_{DP} = profile drag coefficient for wing with blowing
C_{DJ} = jet drag ($=\Delta C_{DP}$)
C_{TR} = jet reaction thrust ($=c_\mu\ \cos\tau$)
C_{DJ_0} = jet drag at zero jet deflection angle
$a(\theta)$ = drag parameter, depending on θ
$a(\alpha)$ = drag parameter, depending on α
C_{D_i}' = induced drag coefficient for wing without blowing
AR = aspect ratio
$a(o)$ = constant
C_2 = constant
C_{D_i} = induced drag coefficient of jet-flapped wing
C_{LR} = jet reaction lift coefficient ($=c_\mu\ \sin\tau$)
K = constant
K' = constant
K'' = constant
ΔC_{TM} = change in total thrust coefficient due to blowing ($\Delta C_{TM} = C_{TM} + C_D' = c_\mu - \Delta C_{DT}$)
ΔC_{DT} = change in total drag coefficient due to blowing ($\Delta C_{DT} = c_\mu - C_{TM} - C_D'$)
ΔC_{LT} = change in total lift coefficient due to blowing ($\Delta C_{LT} = C_{LT} - C_L'$)
ΔC_{DP} = change in profile drag coefficient due to blowing ($=C_{DJ}$)
V_T = takeoff speed of jet flapped aircraft
V_T' = takeoff speed of conventional aircraft
β = angle of climb
τ = jet angle ($=\theta + \alpha$)
δ_f = flap angle

Subscripts
c = calculated
min = minimum

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Introduction

THE once much disputed jet flap thrust hypothesis has been long since verified theoretically Experimentally, however, because of the inherent complexity of experimentally separating thrust and drag, it was as late as 1959 that Foley and Reid[1] announced "the significant finding that substantially complete thrust recovery has been obtained in two-dimensional-flow tests of a jet flap wing " This claim of only "substantially complete thrust recovery" is due to an apparent recovery of only 94% of the jet momentum thrust instead of the 100%, which the much older English[2 3] and French[4] two-dimensional jet flap test results—if plotted in Foley's and Reid's fashion—suggest It will be shown subsequently that also Foley's and Reid's test results, if used correctly, demonstrate that the oblique discharge of air from their jet-flapped wing results in an upstream thrust of magnitude equal to that of the total jet momentum reaction (complete or 100% thrust recovery)

In the scrutiny of existing jet flap test results, one finds that not enough attention has been paid to the application of these results to actual flight vehicles Consequently, basic required data are often lacking This is especially true for information such as how the performance of jet-flapped wings changes as a function of jet deflection angle, angle of attack, rate of blowing, jet flap configuration, aspect ratio, etc To a similar or even greater extent, it applies to data on the economy of jet flap operation such as the expenditure of blowing to achieve a specific lift and the resulting drag penalty Furthermore, it applies to questions of how the jet flap potential as a high-lift device can best be applied to aircraft Both performance and economy of operation are of course vital parameters, for instance, for the design of an STOL-jet flap aircraft Finally, the whole concept of "complete integration of the propulsive system of an aircraft with its lifting system" needs critical examination as to its practical promises This evaluation again strongly hinges on knowing the performance and economy of operation of specifically three-dimensional jet-flapped wings

The basic requirement for all the foregoing problem areas is to find a way of presenting jet flap test results (i e , the lift, drag, jet deflection angle, angle of attack, slot blowing rates and engine thrust, efficiency of lift production, economic operating ranges, etc) in one chart, that is, in a kind of "jet flap characteristics " This is the aim of the work described in this paper

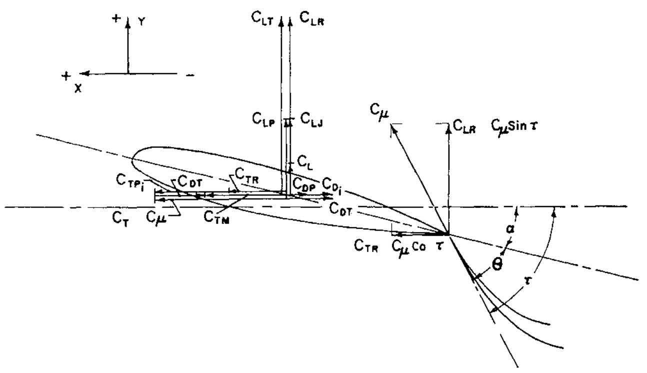

Fig 1 Forces acting on a symmetrical jet-flapped wing at incidence

I Thrust Hypothesis

1 1 Concept

If the full-span jet sheet of a jet-flapped wing in flight is ejected from its trailing edge slot at an arbitrary angle with the wing chord, one might expect only the component of the jet-sheet momentum in thrust direction [the jet reaction thrust C_{TR} (see Fig 1)] to act as thrust However, the thrust hypothesis maintains for two-dimensional jet-flapped wings in ideal fluid flow that the total thrust C_T is equal to the total jet sheet momentum flux c_μ, independent of its angle of deflection θ

In other words, the thrust hypothesis states that, independent of the jet orientation, any increment δc_μ is completely (100%) recovered as thrust $\delta C_T = \delta c_\mu$ Therefore, in inviscid fluid flow, where the total drag C_{DT} is zero, any change in c_μ would result in an equivalent change in the balance-measured thrust C_{TM} (since $C_T = C_{TM}$)

For a viscous fluid flow, the thrust hypothesis can be mathematically expressed as

$$C_{TM} = C_T - C_{DT} = c_\mu - C_{DT} \qquad (1a)$$

Theoretically,[5] the thrust hypothesis was shown to be true Experimentally, because of the inseparability of thrust and drag forces, Eq (1a) cannot be verified by direct measurements, since only C_{TM} and c_μ can actually be measured

1 2 Complete and Incomplete Thrust Recovery

Complete thrust recovery in a real fluid flow would be achieved if c_μ or any change in c_μ, independent of the jet orientation θ, is fully (100%) recovered as thrust or if, in other words, $C_T = c_\mu$ (see Fig 1) Assuming that by experiment this can be shown to be so, any such evidence for 100% thrust recovery would verify the thrust hypothesis experimentally Incomplete thrust recovery ($C_T < c_\mu$) in this context would simply mean that the thrust hypothesis is not true

If, however, the term "thrust recovery" is redefined to mean the recovery of changes δc_μ in the form of actual *balance-measured thrust*, δC_{TM}, this "efficiency of thrust recovery" can be anything from even more than 100% (see Sec 3 1) to zero In the case of a 100% thrust recovery, any increment δc_μ would, independent of jet deflection angle, result in an exactly equal increment δC_{TM} Since this is precisely what the thrust hypothesis predicts to happen in nonviscous flow (where $C_{DT} = 0$), a 100% recovery of any change in blowing (δc_μ) as balance-measured thrust ($\delta C_{TM} = \delta c_\mu$) in an experiment, in which C_{DT} can be shown to be constant, is irrefutable experimental proof of the thrust hypothesis For cases of other than 100% thrust recovery efficiency in which the C_{DT} variation is not definitely known, the thrust hypothesis can be neither proved nor disproved experimentally

1 3 Experimental Proof of Thrust Hypothesis

As discussed in Sec 1 1, the expression that, in concept, represents the thrust hypothesis

$$C_{TM} = c_\mu - C_{DT} \qquad (1b)$$

cannot be proved directly (i e , without inference) by experiment, since only C_{TM} and c_μ can actually be measured This fact led to two avenues of approach for experimental proof of the thrust hypothesis:

1) One accepts the thrust hypothesis as true ($C_T = c_\mu$) and uses the two measured quantities C_{TM} and c_μ to define the total drag C_{DT} as

$$C_{DT} = c_\mu - C_{TM}$$

Then one checks whether the thus-defined experimental C_{DT} makes sense in the light of what one conventionally knows about wing drag

2) One does not accept the thrust hypothesis as true The force balance in the thrust-drag direction then has to be expressed in the more general form

$$C_{TM} = f(c_\mu) - C_{DT}(\theta, c_\mu) \qquad (1c)$$

This expression, if differentiated with respect to c_μ at fixed C_{LT}, becomes

$$(dC_{TM}/dc_\mu)_{C_{LT}=\text{const}} = f' - (dC_{DT}/dc_\mu)_{C_{LT}=\text{const}}$$

Experimental evidence irrefutably proves that $(dC_{TM}/dc_\mu)_{C_{LT}=\text{const}} = 1$ Therefore,

$$f' - (dC_{DT}/dc_\mu)_{C_{LT}=\text{const}} = 1$$

If, in this expression, f' could be shown to be equal to unity, then dC_{DT}/dc_μ would have to be zero Or, if dC_{DT}/dc_μ can experimentally be shown to be zero (C_{DT} = const), then $f' = 1$ [or $f(c_\mu) = c_\mu$], and the thrust hypothesis is verified experimentally

When the first approach is applied to Foley's and Reid's test data,[1] Fig 1 on page 985 of Ref 10 is obtained This figure (for a truly two-dimensional jet-flapped wing) indicates conclusively enough, but not strictly (see deviation of lowest points from straight line), that $C_{DT} = f(C_{LT})$ or that C_{DT} = const C_{LT}^2 That the slope dC_{DT}/dC_{LT}^2 coincides with $dC_D'/dC_L'^2$ is, as shown in Ref 11, a consequence of the infinite aspect ratio, which eliminates the effect of c_μ in the theoretical expression[12]

$$dC_{DT}/dC_{LT}^2 = 1/(\pi AR + 2c_\mu)$$

for three-dimensional jet-flapped wings For quasi-two-dimensional jet flaps ($10 < AR < \infty$), slopes will not necessarily coincide, since the slope dC_{DT}/dC_{LT}^2 decreases with increasing c_μ If Foley's and Reid's experimental range would have included jet deflection angles larger than $\theta = 59\ 1°$ ($\delta_f = 33°$), their test points for these angles would have deviated from the straight line quite appreciably, as demonstrated in Fig 2 for the pure jet-flapped wing ($AR \simeq 20$) tested by the Office Nationale d'Etudes et de Recherches Aeronautiques [4]

When the experimental data of NGTE,[2,3] the Office Nationale d'Etudes et de Recherches Aeronautiques,[4] and Stanford[1] are plotted as C_{TM} vs the real (not calculated) c_μ at fixed C_{LT} values, they were found to fall on straight lines (see, e g , Figs 3 and 4) of slope $dC_{TM}/dc_\mu = 1$ Each such constant C_{LT} line indicates the following:

1) Any change in c_μ produces an exactly equivalent change in C_{TM}

2) Any change in c_μ, independent of the jet orientation, is fully (100%) recovered as balance-measured thrust Since this is exactly what the thrust hypothesis predicts, straight lines of slope $dC_{TM}/dc_\mu = 1$ (C_{LT} = const) constitute convincing experimental proof for the validity of the thrust hypothesis, provided that the total drag along such a straight line can be experimentally shown to be constant (or $dC_{DT}/dc_\mu = 0$)

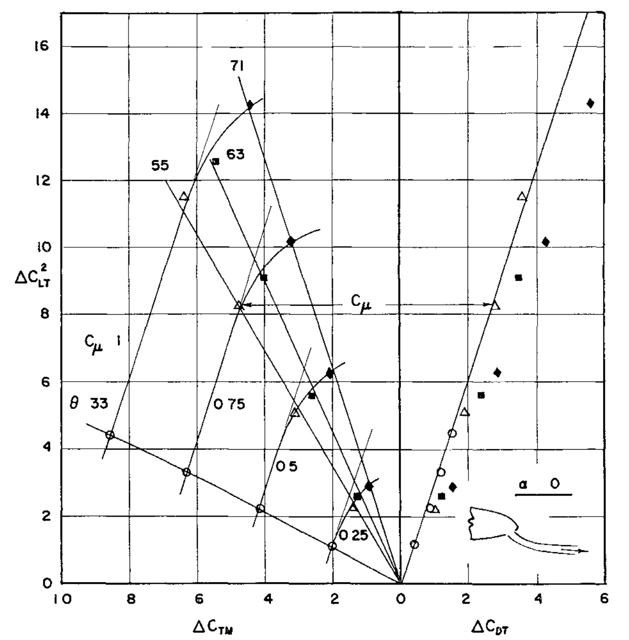

Fig 2 Analysis of thrust recovery (test data for the pure jet flap of Ref 4)

This is strictly true only if C_{DT} is a function of C_{LT} alone, and, if this is the case, C_{DT} plotted vs C_{LT}^2 gives a straight line As will be shown in the next section, this is nearly enough true over significant ranges of θ and c_μ

1 4 Experimental Proof of $dC_{DT}/dc_\mu = 0$ for Complete Thrust Recovery

In Ref 6, the jet flap drag at $\alpha = 0$ was demonstrated to obey the relationship

$$\Delta C_{DT} = a_1(\theta)c_\mu + a_0 c_\mu \tag{2}$$

which, for all $\theta > 0$, can be reduced to

$$\Delta C_{DT} = a(\theta)c_\mu = C_1 \sin^2\theta\, c_\mu \tag{3}$$

where C_1 was found to be constant for $\theta < 60°$

Equation (3) predicts, then, that any changes in ΔC_{DT} caused by changes in either c_μ or θ must eliminate each other, provided that the total lift is kept constant during these changes Since along a ΔC_{LT} = const line the induced drag C_{Di} is constant (assuming that $\pi AR \gg 2c_\mu$, which is justified for quasi-two-dimensional jet-flapped wings), it follows that the profile drag along the straight portion of a ΔC_{LT} = const line must be constant, too This means that all changes in profile drag due to c_μ (defined as "jet drag C_{DJ}") must be counterbalanced by opposite changes in profile drag due to θ, δC_{DP_θ}, such that no net change in profile drag takes place, or

$$\delta(\Delta C_{DP}) = \delta C_{DJ} + \delta C_{DP_\theta} = 0 \tag{4}$$

or

$$-\delta C_{DJ} = \delta C_{DP_\theta} \tag{5}$$

When, however, a ΔC_{LT} = const line starts to deviate from a straight line—a phenomenon that will be discussed later—an increase in total drag results from the fact that the profile drag with increasing jet deflection angles rises faster than the jet drag falls off because of reduced blowing

If Eq (3) is combined with the theoretical expression by Spence,[7]

$$\Delta C_{LT}^2 = K^2 \sin^2\theta\, c_\mu \tag{6}$$

another expression for $\theta > 0$ results as

$$\Delta C_{DT} = (C_1/K^2)\ \Delta C_{LT}^2 \tag{7}$$

where K is given by Spence[7] as

$$K = 2\pi^{1/2} + 0\ 325 c_\mu^{1/2} + 0\ 156 c_\mu \tag{8}$$

which can, as shown in Fig 5, be approximated by

$$K = 3\ 65 + 0\ 35 c_\mu \tag{9}$$

If an average $\bar{K} = 4$ is chosen, the maximum possible deviation of $\bar{K}$ from K over the range of $0 < c_\mu \leqslant 2$ is less than $\pm 10\%$ If C_1 and K are calculated from actual jet flap test results, using Eqs (3) and (6), respectively, it is found[8] that C_1 and K are well-defined constants—C_1 as long as $\theta < 60°$, and K if $c_\mu > 0\ 5$ In other words, over the ranges where C_1 and K are experimentally verified constants,

$$d(\Delta C_{DT})/d\Delta C_{LT}^2 = \text{const} \tag{10}$$

In Fig 6, test data from three different jet-flapped wings confirm Eq (10)

II Jet Flap Characteristics for $\alpha = 0$

2 1 Thrust Recovery and Operating Range

If one plots test data of Refs 1 and 4 over the entire operating range, Figs 7 and 8 are obtained These and plottings of additional data of Ref 4 show that, at large c_μ values and $\theta >$ 50°, the lines of C_{LT} = const become curved and deviate from the lines of constant drag C_{DT} (or 100% thrust recovery) They further show that, the smaller C_{LT} at small c_μ, the

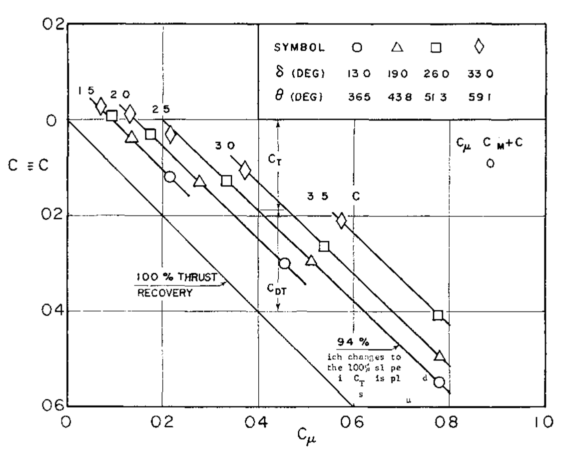

Fig 3 Thrust recovery and thrust and drag variation at constant lift (test data of Ref 1)

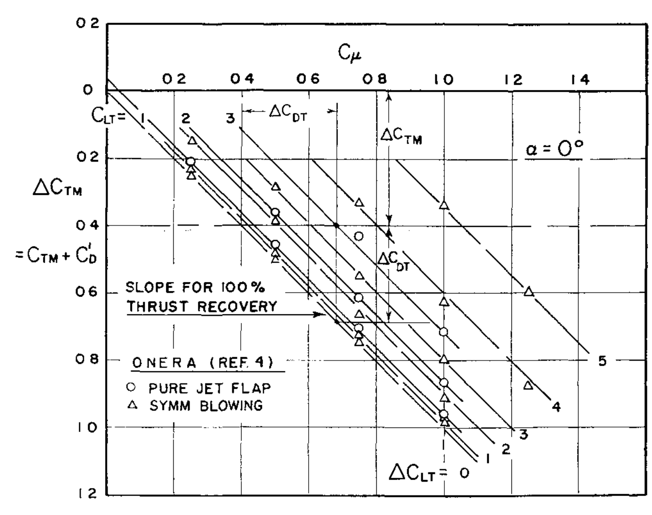

Fig 4 Thrust recovery and thrust and drag variation at constant lift (test data of Ref 4)

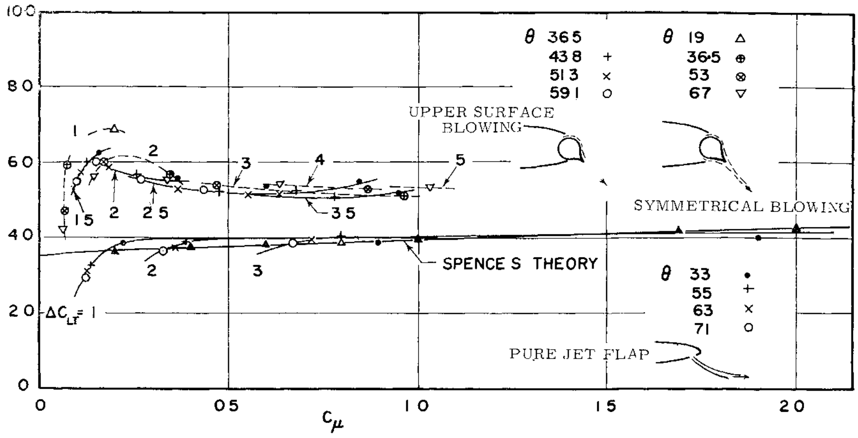

Fig 5 A compaiison of the factoi K as obtained for various jet flap configurations (test data from Refs 1 and 4)

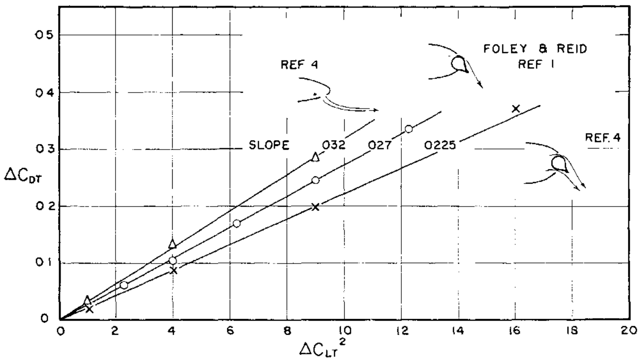

Fig 6 The constant slope $d(\Delta C_{DT})/d(\Delta C_{LT}^2)$ for three different jet-flapped wings

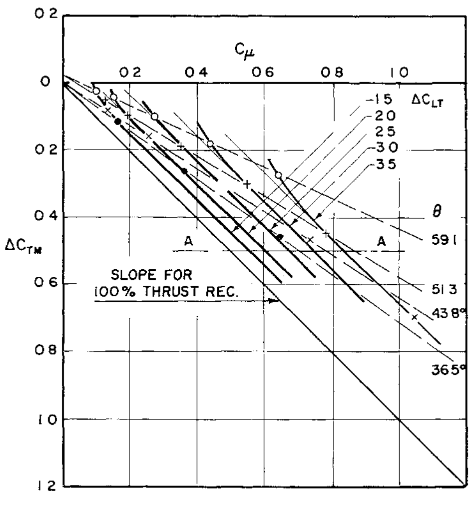

Fig 7 Thrust iecovery and the real thrust and drag variation as a function of lift (corrected test data of Ref 1)

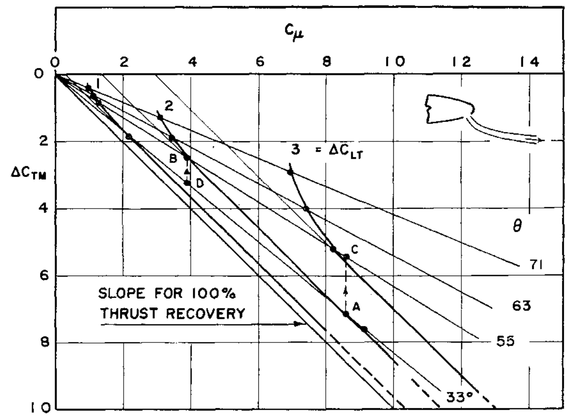

Fig 8 Thrust recovery and the real thrust and drag variation as a function of lift (test data of Ref 4)

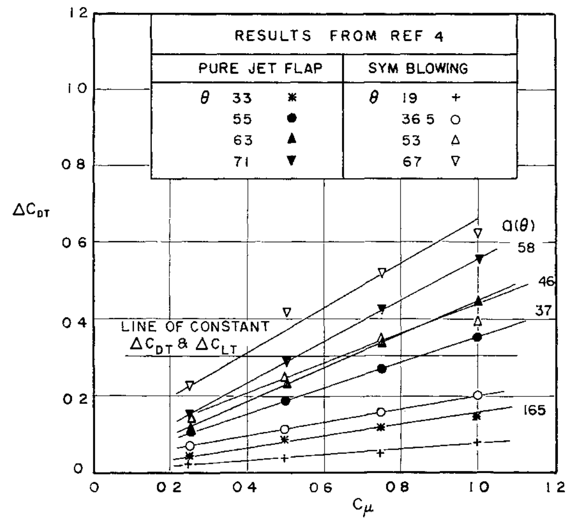

Fig 9 Total drag variation as a function of the rate of blowing (test data of Ref 4)

smaller the jet deflection angle becomes at which a 100% thrust recovery can be secured

The deviation of the ΔC_{LT} = const lines from the straight lines of constant C_{DT} obviously occurs because ΔC_{DT} is increasing instead of remaining constant or because

$$\Delta C_{DT} = (C_1/K^2)\ \Delta C_{LT}^2 \neq \text{const} \tag{11}$$

In other words, C_1 and K can now no longer be constants How C_1 and K are changing is shown for all available quasi-two-dimensional jet-flapped wing test data in Figs 6 and 15 of Ref 8 and Fig 5 of this paper, respectively

Equation (3) states that, for $0 < \theta < 50°$, ΔC_{DT} = const along a ΔC_{LT} = const line, as long as this line is straight Furthermore, lines of constant ΔC_{DT} are straight horizontal lines in, e g , Fig 9, and each of these lines corresponds to a specific lift value, ΔC_{LT}, which can be obtained from

$$\Delta C_{LT} = (K/C_1^{1/2})(\Delta C_{DT})^{1/2} \tag{12}$$

In Fig 10, the test data of Figs 8 and 9 for the pure jet flap of Ref 4 are combined The extent of the straight portions of the ΔC_{LT} = const lines indicates the operational range of constant C_{DT} and of complete thrust recovery; the curved portions indicate where C_{DT} is no longer constant, and changes in c_μ are therefore no longer fully recovered as thrust In this latter range, the large jet deflection angles employed demand

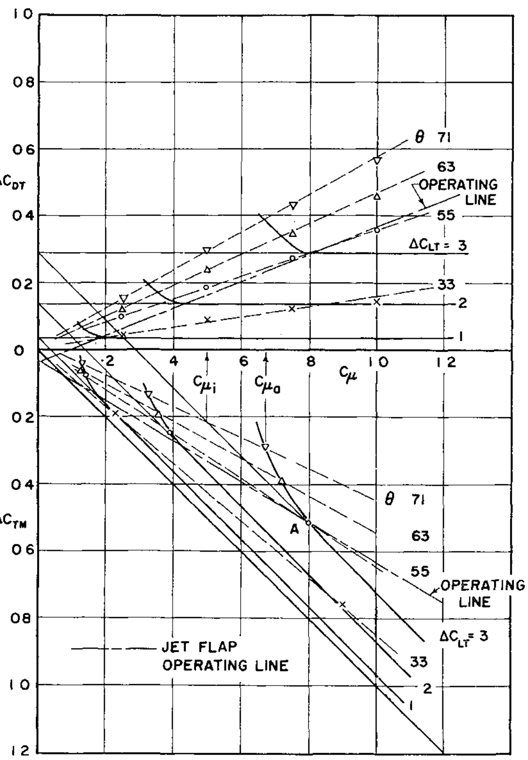

Fig 10 Jet flap characteristics for a pure jet-flapped wing at zero incidence (test data of Ref 4)

stronger blowing in order to compensate for and overcome an increasing, instead of constant, profile drag

2 3 Most Economical Operation

By nature, the jet flap is a high-lift device Its lift is primarily a function of the angle of attack, the jet deflection angle, and the rate of blowing A specific lift is produced most economically if both c_μ and ΔC_{DT} are the smallest values possible

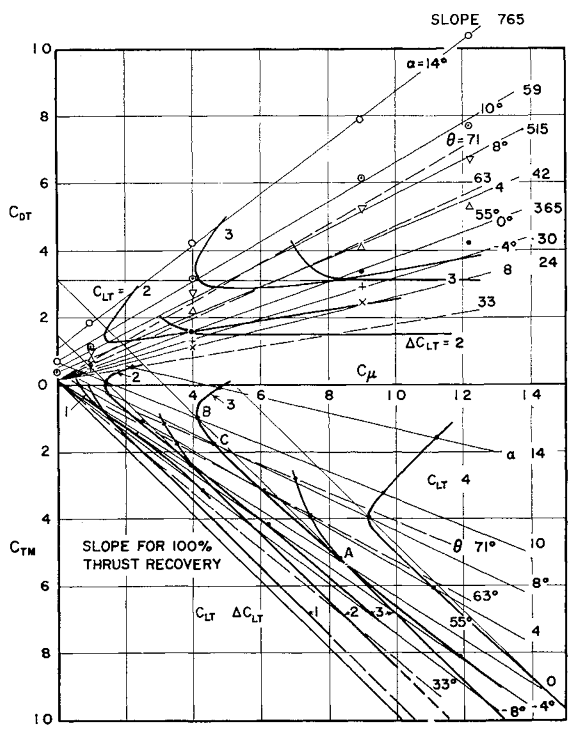

Fig 11 Jet flap characteristics for a pure jet-flapped wing at θ = 55° and varying incidence (test data of Ref 4)

If one operates, e g , to the right of point A (see Fig 10) along the ΔC_{LT} = 3 line, operation is uneconomical because the lift is primarily due to blowing without making the fullest use of the potential of the jet deflection angle as a means for producing lift Any lift increase due to θ is free of charge, as long as the total drag does not increase (100% thrust recovery) and leads to an appreciable reduction in the amount of blowing required to produce it Operation, however, to the left of A along ΔC_{LT} = 3 indicates that, e g , at θ = 71°, the actually required $c_{\mu a}$ = 0 67 as compared with $c_{\mu i}$ = 0 49, which is the ideal jet coefficient at θ = 71°, if the thrust recovery is still 100%

Note that at low ΔC_{LT} values the range of θ, over which the total drag does not change, is rather small (for ΔC_{LT} = 1, $0 < \theta < 33°$) However, for such small lift coefficients ($\Delta C_{LT} < 1$ 5), one would not employ jet-flapped wings anyway Note, too, that at high ΔC_{LT} values θ should not, it seems, exceed 60° if complete thrust recovery is to be secured

For most economical lift production, jet-flapped wings should be operated along the "operating line" (see Fig 10), which is obtained by connecting the points at which the ΔC_{LT} = const lines start to deviate from the straight lines of constant C_{DT} These points correspond to the lowest possible c_μ and C_{DT} values at which a specific lift can be produced Note that this operating line is about parallel to the θ = 63° line This suggests that jet deflection angles greater than 60° should not be used if highly economical operation of this jet-flapped wing is desired

III Jet Flap Characteristics for $\alpha \neq 0$

Unfortunately, there is only one set of quasi-two-dimensional test results available (Office Nationale de'Etudes et de Recherches Aeronautiques[4]) which presents the drag of a pure jet-flapped wing as a function of the wing's angle of attack The jet deflection angle in these tests is kept constant at θ = 55°

3 1 Evaluation of Test Data

The test results of Fig 9 of Ref 4, replotted in Foley's and Reid's fashion, are shown in Fig 11 Note the strong influence of α on the curvature of the C_{LT} = const lines, especially at high C_{LT} values Here, because of wing stall, α can no longer keep the lift at a constant value when c_μ is decreasing At angles of attack approaching stall, at stall and beyond, c_μ must finally increase again to compensate for the loss in lift increase with increasing α

It is obvious that it would have been impossible to deduce unambiguous confirmation of the thrust hypothesis from Fig 11 Only in the range of $\alpha = \pm 4°$ do the constant lift lines approach the 100% thrust recovery slope This is because over this α range (at constant θ) the profile drag is practically constant This is a prerequisite for the condition of constant jet flap drag, which must be fulfilled if complete thrust recovery is to be obtained along ΔC_{LT} = const lines

Figure 10 demonstrates that decreasing c_μ along the C_{LT} = 3 lines past point A, for instance, would require an increase of the jet deflection angle to values $\theta > 55°$ Since the C_{LT} = 3 line above point A starts to deviate from the C_{DT} = const line, any increment δc_μ has to accommodate a drag increment δC_{DT}, in addition to the thrust change δC_{TM}, or

$$\delta C_{TM} = \delta c_\mu - \delta C_{DT} \qquad (13)$$

If, however, instead of θ, α is increased (at θ = 55°), one sees that the lift coefficient of 3 can be maintained at no increase in total drag down to very appreciably lower rates of blowing (see point B) The very best jet flap operating point actually would be at C, where the desired lift (C_{LT} = 3) is obtained at the lowest possible total drag The reason for the curve ACB being located below the C_{DT} = const or 100% thrust recovery line through point A is as follows: when at

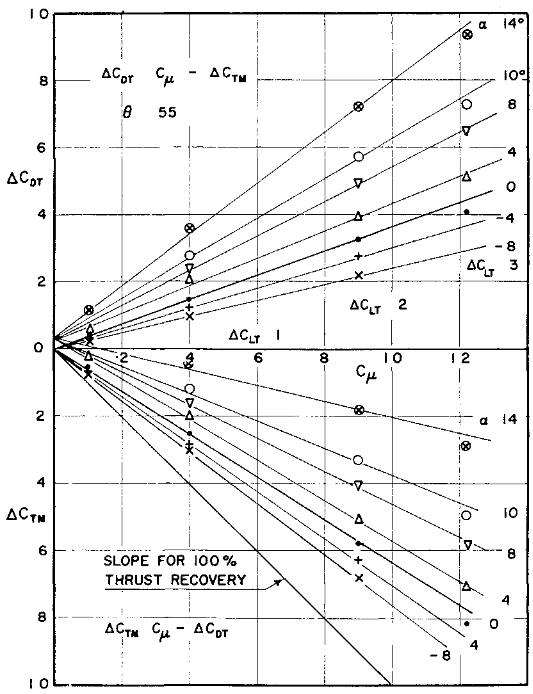

Fig 12 Variation of total drag and thrust of a pure jet-flapped wing due to blowing and angle of attack (test data of Ref 4)

point $A(\theta = 55°, \alpha = 0°)$ α is increased, the lift component of c_μ is increased due to the rotation of the c_μ vector by an angle α This, because of the condition of $C_{LT} = \text{const} = 3$, requires c_μ to be reduced, whereby simultaneously the drag of the jet-flapped wing is reduced (see, e g , Fig 9) Moreover, the lift is also enhanced on account of putting the wing at α, and elimination of this lift increase (in order to keep C_{LT} constant at a value of 3) leads to a further drag reduction However, besides increases in lift, α, in general, also causes the drag to rise But this drag increase is small as long as α is small ($\alpha < 8°$) The integrated effect that the angle of attack of a jet-flapped wing at fixed C_{LT} has on the flow field surrounding the wing leads to a reduction in total drag Therefore, the line ACB is below the line of $C_{DT} = \text{const}$ (or 100% thrust recovery efficiency) Thrust recovery efficiencies for points on the line ACB are obviously higher than 100% This fact reflects on the relative usefulness of this efficiency which, as was pointed out in Sec 1 2, makes sense only in cases of complete (100%) thrust recovery Beyond point B, the drag increase due to α approaching stall dominates the drag-thrust balance

3 2 Most Economical Operation at Angles of Attack

Figure 11 demonstrates that, for most economical operation of jet-flapped wings, the use of the angle of attack is as important a means as the jet deflection angle

In general, jet-flapped wings operate most economically if they produce a desired lift at the lowest possible drag and for the least amount of blowing For instance, for a lift coefficient $C_{LT} = 3$, point C of Fig 11 meets these requirements, and the optimum values of θ and α seem to be 60° and 8°, respectively A line of most economical operation or an "operating line" could be added (omitted here to avoid overcrowding) which is the locus of points at which the $C_{LT} = \text{const}$ lines for changing α intersect (see, for instance, point B) the $C_{DT} = \text{const}$ or 100% thrust recovery line Better still would be a line that connects the points where tangents parallel to the 100% thrust recovery lines touch the $C_{LT} = \text{const}$ lines for changing α (see, for instance, point C on curve ACB)

In comparing the operating lines of a jet-flapped wing at zero angle of attack (see Fig 10) with the one just discussed, the reductions in blowing rates due to α are quite appreciable (50% for $C_{LT} = 3$; see point A and B of Fig 11) If the engine thrust coefficient $c_{\mu T}$ is smaller or equal to the jet momentum coefficient $c_{\mu L}$ required for the production of the desired lift, the use of α is essential If $c_{\mu T}$ is greater than $c_{\mu L}$ at point B but smaller than the amount of blowing required at zero α and optimum θ (point A), the full use of the angle of attack is still advantageous, especially if only that portion of the jet engine exhaust is ejected through the trailing edge slots which is required solely for the production of the desired lift If, however, $c_{\mu T}$ is greater than the $c_{\mu L}$ required at zero α (point A), beneficial effects from the use of the angle of attack can be derived only if the jet engine exhaust through the wing trailing edge slots is limited strictly to that needed for producing the desired lift The benefits in this case are indirect and concern pressure losses, structural weight, and cross-sectional areas of the ducting from the engine to the trailing edge slots It may well be that these gains do not justify the complications of the takeoff maneuvers which are imposed on the pilot as a result of the use of α in addition to the jet deflection angle

3 3 Total Drag as a Function of c_μ

If one plots the total drag vs c_μ for the pure jet-flapped wing of Ref 4 at $\theta = 55°$ with α as the parameter, Fig 12 is obtained We see that, as in Fig 9, the total drag obeys a relationship of the form

$$\Delta C_{DT} = a(\alpha)c_\mu \qquad (14)$$

at least as long as $c_\mu < 1\ 2$ Obviously, per degree ΔC_{DT} increases more strongly with α than with θ

If we plot $a(\alpha)$ vs $\sin^2\alpha$, the resulting curve (see Ref 8) suggests that, for the range $0° < \alpha < 10$,

$$d[a(\alpha)]/d \sin^2\alpha = C_2 \qquad (15)$$

where C_2 for this particular pure jet flap was found to be a constant equal to 7 7 From Eq (15), it then follows that

$$a(\alpha) + C = C_2 \sin^2\alpha \qquad (16)$$

Since, for $\alpha = 0$, $C = -a(0)$, we get

$$a(\alpha) = a(0) + C_2 \sin^2\alpha \qquad (17)$$

and

$$\Delta C_{DT} = [a(0) + C_2 \sin^2\alpha]c_\mu \qquad (18)$$

Since $\Delta C_{TM} = c_\mu - \Delta C_{DT}$, it follows that

$$\Delta C_{TM} = c_\mu\{1 - [a(0) + C_2 \sin^2\alpha]\} \qquad (19)$$

for $0° < \alpha < 10°$

Note, that in Fig 11 the line through A and the origin represents the line $\theta = 55°$ and the line $\alpha = 0°$ The total drag change along this line can therefore be calculated from either Eq (3) or Eq (18), which means that

$$C_1 \sin^2\theta = [a(0) + C_2 \sin^2\alpha] \qquad (20)$$

and

$$a(0) = a(\theta) \qquad (21)$$

since $\alpha = 0$, or

$$\Delta C_{DT} = [a(\theta) + C_2 \sin^2\alpha]c_\mu \qquad (22)$$

3 4 Construction of a Jet Flap Characteristics

Below the operating line of a characteristics, where along $C_{LT} = \text{const}$ lines the total drag is constant and the thrust recovery is complete, the values of C_1, C_2, and K were found[8] to be constants If these constants are known for a particular jet-flapped wing, its characteristics can be artificially constructed, or an existing incomplete characteristics can be supplemented

Pick, e g , point A in Fig 11 Its total drag follows from Eq (3) or (18) and its total lift from Eq (7) Inserting any other lift value into Eq (7) furnishes the corresponding total drag and permits one to add this C_{LT} = const line to the characteristics Similarly, one can add lines of constant θ or α If any θ and c_μ values are selected at $\alpha = 0$, the total drag follows from Eq (3); if α and c_μ are changed at $\theta = 55°$, the total drag is obtained from Eq (22) In this way, that portion of a jet flap characteristics which represents the region of most economical jet flap operation can be artificially constructed or supplemented

IV Jet Flap Application to Aircraft

4 1 Jet-Flapped Wing and STOL

This consideration is rather hypothetical on account of the high aspect ratio ($AR \simeq 20$) of the jet flapped wing represented in the characteristics of Fig 11 Nevertheless, if later a truly three-dimensional jet flap is similarly evaluated, the effect of aspect ratio on the STOL potential of jet-flapped wings can be deduced

4 1 1 Lift analysis

In Ref 9, averaged thrust coefficients at takeoff and cruise of fighter aircraft, airliners, and trainers were found to be as follows: at takeoff, $c_{\mu T} = 0\ 5$; at cruise, $c_{\mu T} = 0\ 025$

Let us now imagine that a conventional aircraft is to be converted into a STOL aircraft by means of the jet flap principle The takeoff run is to be shortened to $\frac{1}{6}$ of its conventional takeoff distance Weight and thrust are assumed to be the same for both aircraft Since the lift force at takeoff must be the same for both aircraft, it follows that

$$C_L'(\rho/2)\ V_T{}'^2\ S_W = C_{LT}(\rho/2)\cdot V_T{}^2\ S_W = C_{LT}(\rho/2)\ (V_T{}'^2/6)\ S_W$$

assuming constant acceleration Consequently, $C_{LT} = 6C_L'$, and $c_\mu = 6c_{\mu T} = 3$ For $C_L' = 1\ 2$, the required takeoff lift C_{LT} would then have to be $C_{LT} = 7\ 2$

If the pure jet flap of Fig 11 would be used at $\theta = 60°$, $\alpha = 0°$, and $c_\mu = 3$, C_{LT} would be found from Eq (6) as

$$\Delta C_{LT} = C_{LT} = (4\ 1^2\ 0\ 75\ 3)^{1/2} = 6\ 15$$

Obviously, this lift would be too small to get the aircraft off the ground at the prescribed takeoff point However, the lift could be increased to $C_{LT} = 7\ 2$ either by increasing the engine thrust in order to raise c_μ from 3 to 4 1 (a 37% thrust increase) or by choosing a jet-flapped wing, which at $\theta = 60°$, $\alpha = 0$, and $c_\mu = 3$ produces a higher lift Such aero foils are those with jet control flaps, and their higher lift values are due to a higher K (see Fig 4) The K required here follows from Eq (6) for $C_{LT} = 7\ 2$ as $K = 4\ 8$ Either wing in Fig 5, i e , the one for upper surface or the one for symmetrical blowing over the jet control flap, would be suitable

If the angle of attack, in addition to $\theta = 60°$, would be used for lift production, the required $c_\mu = 4\ 1$ for the pure jet-flapped wing of Fig 11 could be reduced, it seems, to about half the preceding value This leaves two alternatives: to eject the entire engine mass flow through the trailing edge slots of the wing irrespective of how much of it is actually required for lift production, or to exhaust only that portion of the engine mass flow through the slots which produces the required lift The remainder of the engine exhaust, and, eventually during cruise, the entire engine mass flow, is ejected in the conventional way through the engine's exhaust nozzle

4 1 2 Thrust Analysis

At the takeoff point, besides lift, enough thrust has to be supplied to permit the desired rate of climb How much thrust is available for climb depends on the total drag of the jet flap version at takeoff This total drag can be calculated for the jet-flapped wing of symmetrical blowing (see Fig 6) as

$$C_{DT} = 0\ 0225\ 7\ 2^2 = 1\ 18$$

or

$$C_{TM} = 3 - 1\ 18 = 1\ 82$$

The maximum angle of climb at constant takeoff speed V_T follows, then, from

$$\tan\beta = 1\ 82/7\ 2 \text{ as } \beta \simeq 14°$$

A 50-ft obstacle could be cleared at a distance of approximately 250 ft, including a 50-ft transition region Shorter distances would require corresponding increases in engine thrust Note that employment of α for lift production does not change the total drag ($C_{DT} = 1\ 7$) as long as one does not operate above the operating line

V Conclusions

Presented jet flap characteristics indicate the following:

1) The total drag and profile drag remain constant along the straight portion of any constant lift line (100% thrust recovery)

2) The extent of the straight portions of the constant lift lines defines the range of economical jet flap operation The locus of points where the constant lift lines deviate from a 100% thrust recovery line define the jet flap operating line of optimum jet flap economy

3) The jet flap operating line confines the jet deflection angle to $\theta < 60°$ and the angle of attack to $\alpha < 8°$

4) The angle of attack, besides θ, is a very potent means of improving the economy of operation of high-aspect ratio jet flapped wings

5) That portion of a jet flap characteristics which represents the practical operating range of a jet flap can be constructed artifically if the "constants" C_1, C_2, and K are known from tests

The operation of jet flapped wings along the operating line for both optimum performance and economy is practically feasible only if merely that portion of the jet engine mass flow is ejected through the wing trailing edge slots which is required for the production of the desired lift In practice, depending on the wing's aspect ratio, this portion may be smaller than or equal to the engine mass flow This question and its repercussions on the design of jet flap STOL aircraft will be discussed in a forthcoming report presenting the characteristics of three-dimensional jet flapped wings

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Effect of Microwave Radiation on the Ionized Gas behind a Strong Normal Shock Wave

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An analytical study of the interaction between microwaves and a fully ionized gas behind a normal shock wave is presented The governing differential equations derived by a quasi-steady analysis aıe used to obtain the nondimensional parameters and integrated to yield the "first integrals" of the momentum and energy equations A modified Prandtl relation is derived from which the final equilibrium state of the ionized gas can be found Ionization is seen to increase the final equilibrium density, whereas microwave heating tends to reduce its value It is further shown that thermal choking of the ionized gas occurs at a critical microwave power level, which is a function of the Mach number ahead of the shock wave, the reflection coefficient of the microwave, and the ionization potential expressed in terms of the neutral gas temperature ahead of the shock wave At higher power levels, the shock wave moves upstream allowing additional microwave heating of the ionized gas A formula is given for the determination of this equivalent higher Mach number At a sufficiently high power level (but slightly below the critical value), the mean gas tempeıature is shown to attain a peak value higher than its final equilibrium value This peak temperature actually represents the upper bound of the mean gas temperature for a given initial Mach number

I Introduction

IN flowing through a strong normal shock wave, a neutral gas can be ionized When microwaves are directed at the ionized gas along the flow direction, interaction of the ionized gas with the microwaves will occur The present study of the interaction problem will be concerned with the determination of the thermodynamic state (pressure, density, and temperature) the flow state (velocity) of the gas, and the reflection and propagation of the microwaves In view of the well-known thermal choking phenomenon, it is of interest to determine the microwave power level at which the choking will occur and to see what happens to the flow at a higher-power level

Evidently, this mathematical problem is a very difficult one Even the simpler problem of microwave propagation in a medium with fixed nonuniform electron distribution is known to be a formidable one and few exact solutions are available Therefore, in the present study, only the case where complete, or nearly complete ionization is achieved by the shock wave is considered By using the conservation laws and certain integral relations, it is shown that the final equilibrium state of the gas can be determined without difficulty In addition, some physical features of the interaction problem are discussed in some detail

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II Basic Equations—Nondimensional Parameters

The incident microwave field is assumed to be plane polarized (in x-y plane) and is of the form $E_1 \exp\{i(\omega t - kx)\}$, where E_1 is a real constant, and no steady electric or magnetic field is present Propagation of microwaves into a nonuniform plasma generally gives rise to reflected waves ahead of the shock wave of the form $E_1\tilde{R} \exp\{i(\omega t + kx)\}$, where $\tilde{R} = R \exp(i\theta)$, R being the reflection coefficient and θ the phase shift angle The transmitted waves $E(x) \exp(i\omega t)$ will be gradually damped out by the finite resistance of the ionized gas, and a portion of the incident microwave energy will appear as thermal energy and kinetic energy of the gas These are the main features of the interaction between the microwaves and the ionized gas Others such as the possible excitation of the longitudinal electrostatic waves and higher harmonics of the transverse electromagnetic waves will not be considered

To treat the interaction problem, the microwaves and the flow of the ionized gas must be considered as one "system" In a quasi steady state, the physical quantities of the system may be conveniently separated into time-averaged (or